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3D Necker Cube

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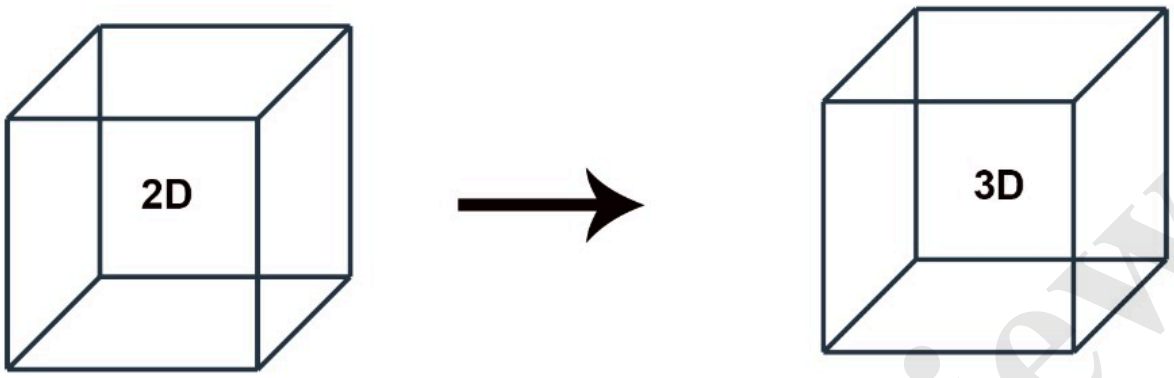
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Building a 3D Necker Cube

A REAL THREE-DIMENSIONAL OBJECT

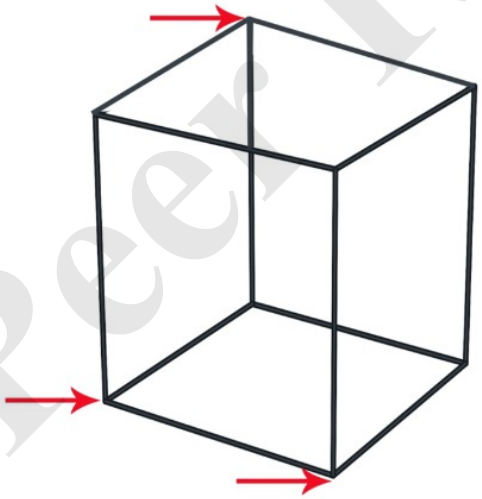


THE CLASSIC NECKER CUBE

A two-dimensional drawing, two possible interpretations.

THE 3D NECKER CUBE

A three-dimensional form, two possible interpretations.



TRUNCATED PYRAMID

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THE PROBLEM

When a regular wireframe cube is viewed with one eye, two interpretations may alternate: one is cubical; the other takes the form of a truncated pyramid [2].

The object remains regular, but its perceived shape can change when depth reverses. What geometry could preserve two cubical interpretations as the object rotates or the observer moves?

Why does the cube seem distorted?

The object does not change. The depth we assign to it can reverse.

A cube, then a truncated pyramid

Viewing a real wireframe cube with one eye, Bertamini and colleagues described alternating perceptions of a cube and a truncated pyramid [2]. The object remains unchanged; reversing its apparent depth changes the shape we recognize.

Perspective preserves relative image sizes

Perspective makes distant parts appear smaller. After a depth reversal, those same parts may seem close, although their sizes in the image remain unchanged. This reversal can make the cube appear irregular [2]. An oblique view should therefore not be confused with a perceptual reversal of depth.

From a known phenomenon to a design problem

As early as 1964, Shopland and Gregory studied visual reversals of a three-dimensional cube: touch made them less frequent without eliminating them [3]. My work approaches this question through construction. I seek a geometry that retains a cubical appearance in both interpretations, including during movement.

For Peer Review

Scientific references

[2] Bertamini, M., Masala, L., Meyer, G. & Bruno, N. (2010). Vision, haptics, and attention: New data from a multisensory Necker cube. Perception, 39(2), 195–207.
https://doi.org/10.1068/p6461

[3] Shopland, C. & Gregory, R. L. (1964). The Effect of Touch on a Visually Ambiguous Three-dimensional Figure. Quarterly Journal of Experimental Psychology, 16(1).
https://doi.org/10.1080/17470216408416348

From an ambiguous drawing to a real object

Necker’s drawing fits on a sheet of paper. My work begins when we try to give it volume and walk around it.

Choose a face of the model and imagine it at the front. Then try to see it at the back. If the depth reverses, the object itself has not moved.

I explored this shape in my workshop by building and modifying the model. A convincing view was not enough: I wanted to change position and still recover both interpretations.



Figure 1. Physical model on its support. Photograph: Olivier Redon.

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Two distinct requirements

The challenge is not simply to produce a depth reversal. Each interpretation must also look like a cube. These two goals guide my experiments: making the volume perceptually reversible while preserving its apparent shape.

Movement adds another requirement: preserving both interpretations as the viewpoint changes.

Which face is in front?

A flat image can suggest a volume whose front and back seem to exchange places. Each interpretation may remain stable for a while: this is bistability. In 1832, Necker described this ambiguity in a drawing of a crystal [1].

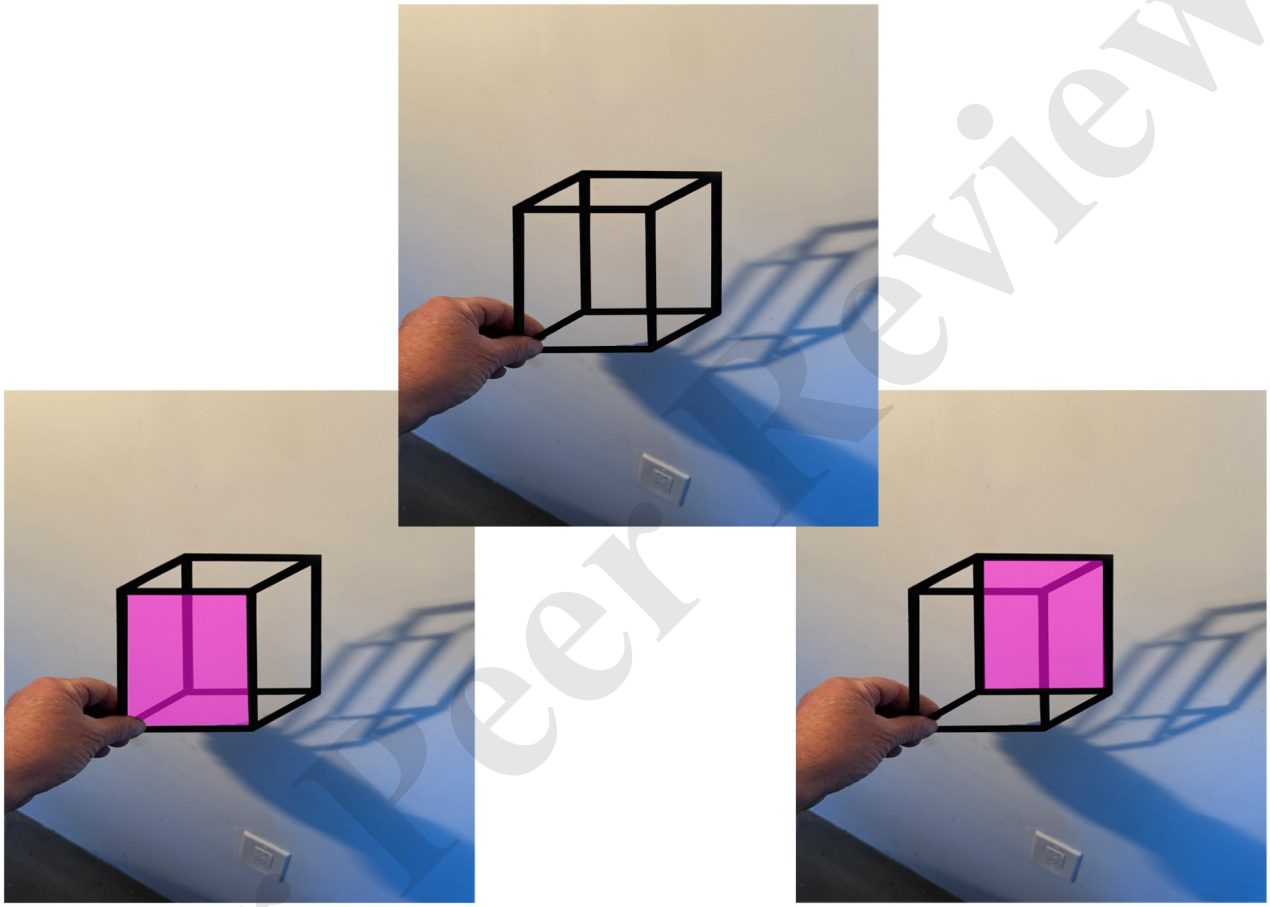


Figure 2. Top: the unmarked structure. Bottom: two possible front faces. The pink areas are graphic guides, not panels added to the object. Montage: Olivier Redon.

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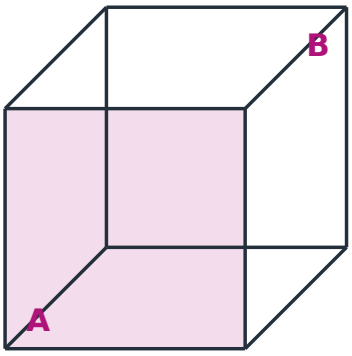
Try switching interpretations

Choose a front face in the uncolored view. Compare the two marked views, then return to the first image: can you now see that face at the back?

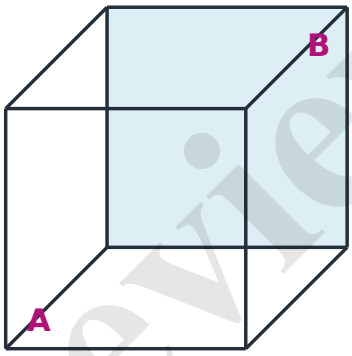
Do not force the reversal if it does not happen immediately. Let your gaze move between the corners, then return to the drawing after reading.

One drawing, two depths

In space, a cube has eight vertices, twelve equal edges and six square faces. These properties do not carry over unchanged onto the page: the drawn lengths and angles depend on how the solid is represented.



Interpretation 1: A in front of B

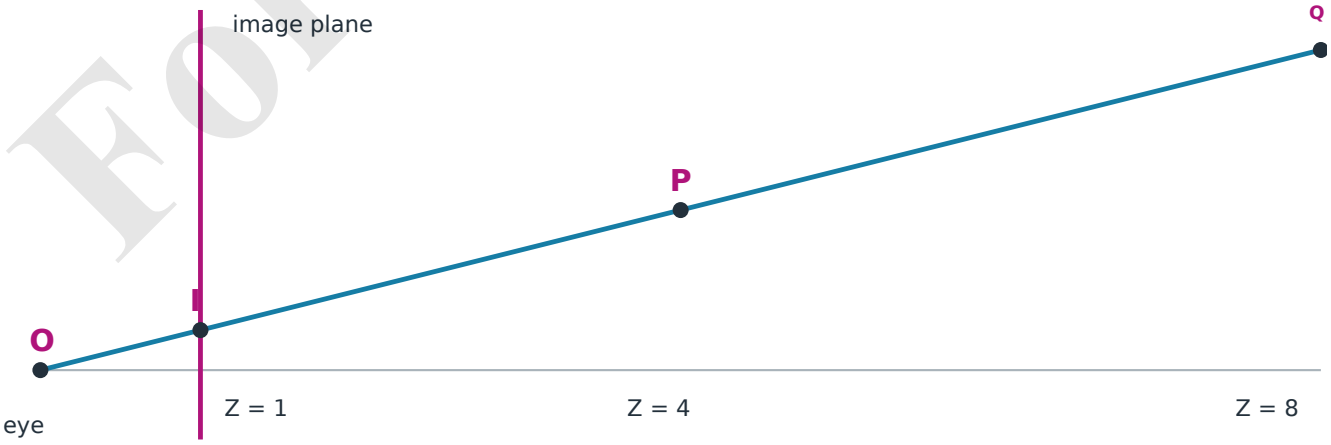


Interpretation 2: B in front of A

The edges occupy exactly the same positions in both drawings. Color simply encourages you to choose a different front face. Mentally exchanging front and back changes the perceived depth without changing the drawing.

A and B identify the two squares in this diagram. They are not the construction labels used for the photographed model.

A direction does not determine a distance



Top view. P = (1, 0, 4) and Q = (2, 0, 8) project onto the same point I: 1/4 = 2/8. They share a line of sight but lie at different depths.

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Perspective in two equations

Close one eye and align your fingertip with a detail on a wall. They lie at different distances but share a line of sight. Move your head slightly: the alignment disappears. The following equations describe this experience.

Projecting a point in space

Place an ideal camera at O = (0, 0, 0), with an image plane one unit in front of it. For a point P = (X, Y, Z), X gives its horizontal position, Y its height and Z its depth. Use the same unit throughout, and choose Z > 1.

$$x = X / Z \text{ and } y = Y / Z$$

The same ratio can correspond to different distances. The points (1, 1, 4) and (2, 2, 8) both project onto (1/4, 1/4). More generally, multiplying all three coordinates by the same positive number moves the point along its line of sight without changing its image.

Moving the viewpoint

If the camera moves t units to the right without rotating, a fixed point appears at x(t) = (X − t) / Z. Its change per unit of camera displacement is therefore:

$$dx / dt = -1 / Z$$

The minus sign indicates an apparent shift to the left. At depth 4, the point shifts twice as fast as at depth 8. This difference provides a distance cue: motion parallax.

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Building a cubical appearance

Paradoxically, I built a truncated pyramid so that it would be perceived as a cube. Rather than preserving the object's geometric regularity, I altered its structure to seek a fully cubical appearance in each of the two interpretations.

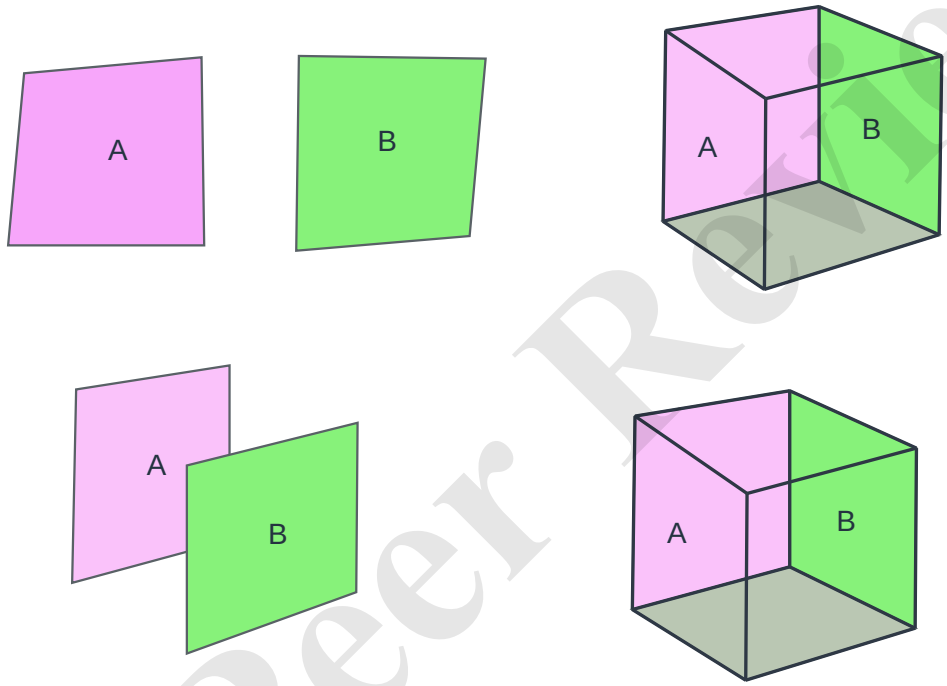


Figure 3. Construction diagram without dimensions, redrawn from Olivier Redon's document. Colors distinguish faces and groups of edges.

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Putting the geometry to the test

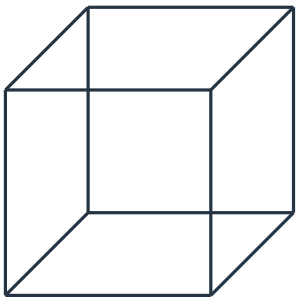
Faces A and B help you follow the assembly. Compare their separate shapes with their positions in the volume: moving from individual parts to the whole makes the construction approach easier to understand.

The decisive test takes place in front of the object. Change position, then rotate it: are both interpretations still accessible, and does each still give you the impression of seeing a cube?

Recreating and observing the effect

1. Draw the ambiguous figure

Draw a square with 6 cm sides, then an identical square shifted 2.5 cm to the right and 2.5 cm upward. Connect corresponding corners with lines of equal thickness, without shading or dashed edges.



This drawing uses an oblique projection: parallel edges remain parallel. It does not reproduce a camera's perspective exactly.

Imagine each square in turn in front of the other. The colored views on page 5 may help.

2. Compare stillness and movement

Place the model on a stable support, keeping viewing distance and lighting constant. Observe it for 30 seconds with one eye, then with both eyes. Note which face seems closest and any depth reversals.

Next, move your head sideways over about 20 cm. In a separate trial, keep your head still and rotate the object slowly. Distinguish depth reversal, apparent rotation and changes in perceived shape in each trial.

These durations and distances are practical starting points, not validated conditions for this model.

3. Distinguish a diagram from a building plan

The illustration on page 7 shows how the faces and edges are arranged, without dimensions. It explains the construction principle but is not a complete fabrication template.

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Preserving ambiguity during movement

A shape may look convincing from the front and lose its ambiguity from the side. Moving between viewpoints reveals how the model behaves.

Reversible outlines, coherent interpretations

For each diamond-shaped outline, I seek two possible orientations in depth. This reversibility must also hold across the entire volume: the interpretations of the different faces must agree to form one coherent cube.

The same principle guides the movement tests. I seek to preserve this agreement as the object rotates or the observer moves. The precise range of positions in which both interpretations persist remains to be established.

Why does a stationary cube seem to rotate?

As we move, parallax changes the cube's image. If its perceived depth remains reversed, visible displacements no longer match apparent distances. Attributing rotation to the object could reconcile these cues. This is a hypothesis; its role in my model still needs testing.

What to distinguish when observing

Distinguish three observations: depth reversal, preservation of a cubical shape and apparent rotation. Also examine shadows, edge thickness and the support. These details may reveal the true depth and break the ambiguity.

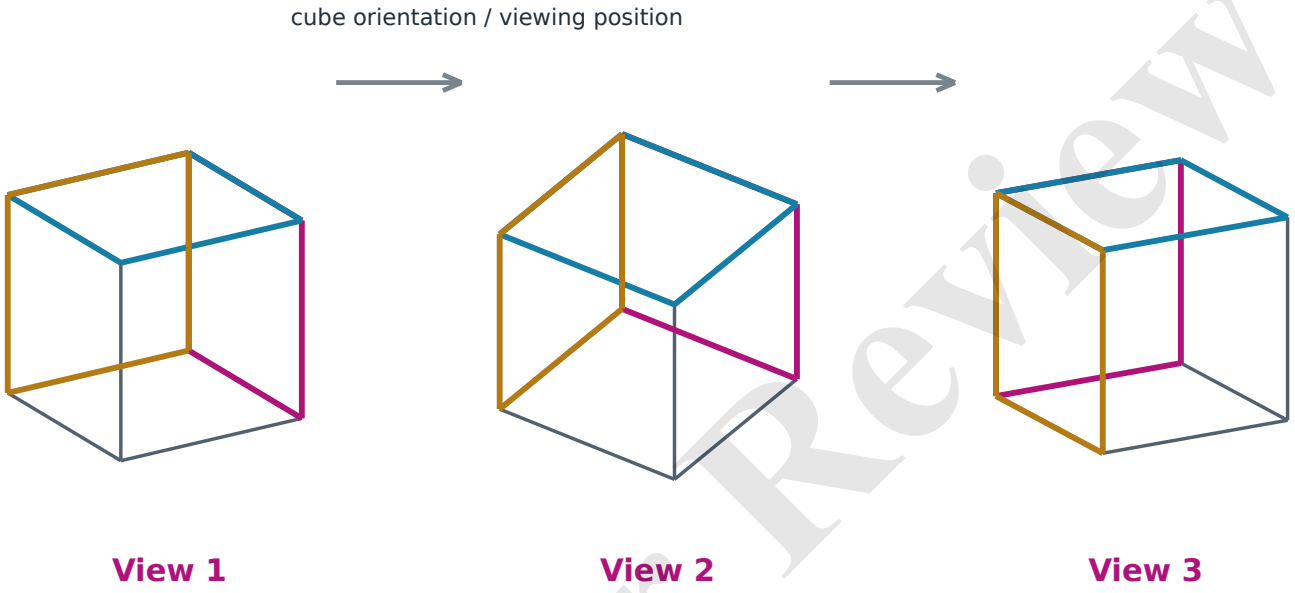
Historical reference

[1] Necker, L. A. (1832). Observations on some remarkable optical phænomena seen in Switzerland; and on an optical phænomenon which occurs on viewing a figure of a crystal or geometrical solid. Philosophical Magazine, 1(5), 329–337.
https://doi.org/10.1080/14786443208647909

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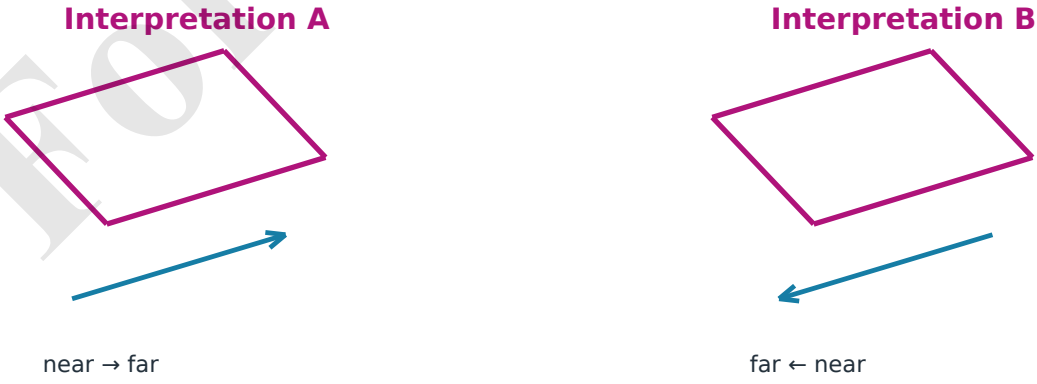
Two interpretations of each outline

For the volume to remain ambiguous, making one face reversible is not enough: both interpretations must remain coherent throughout the structure.



Colors identify three faces. In these views, their outlines are parallelograms, which I describe visually as diamonds. The same reasoning applies to the opposite faces when a change of orientation brings them into view.

The same outline, reversed depth

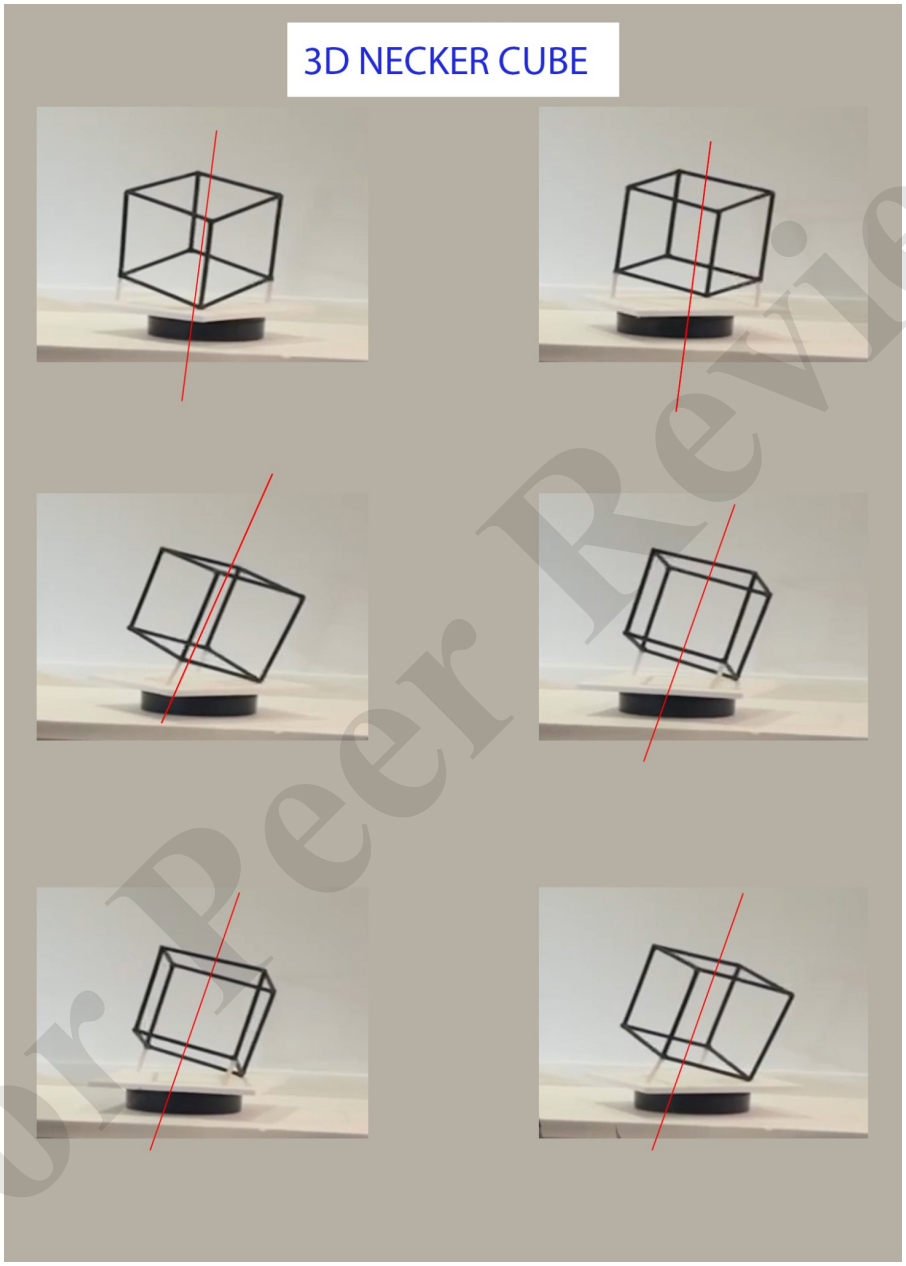


Conceptual diagram, not a dimensioned plan. These views explain the design criterion; they do not demonstrate that it holds in every position.

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The cube at different orientations

Original document A01 · Olivier Redon



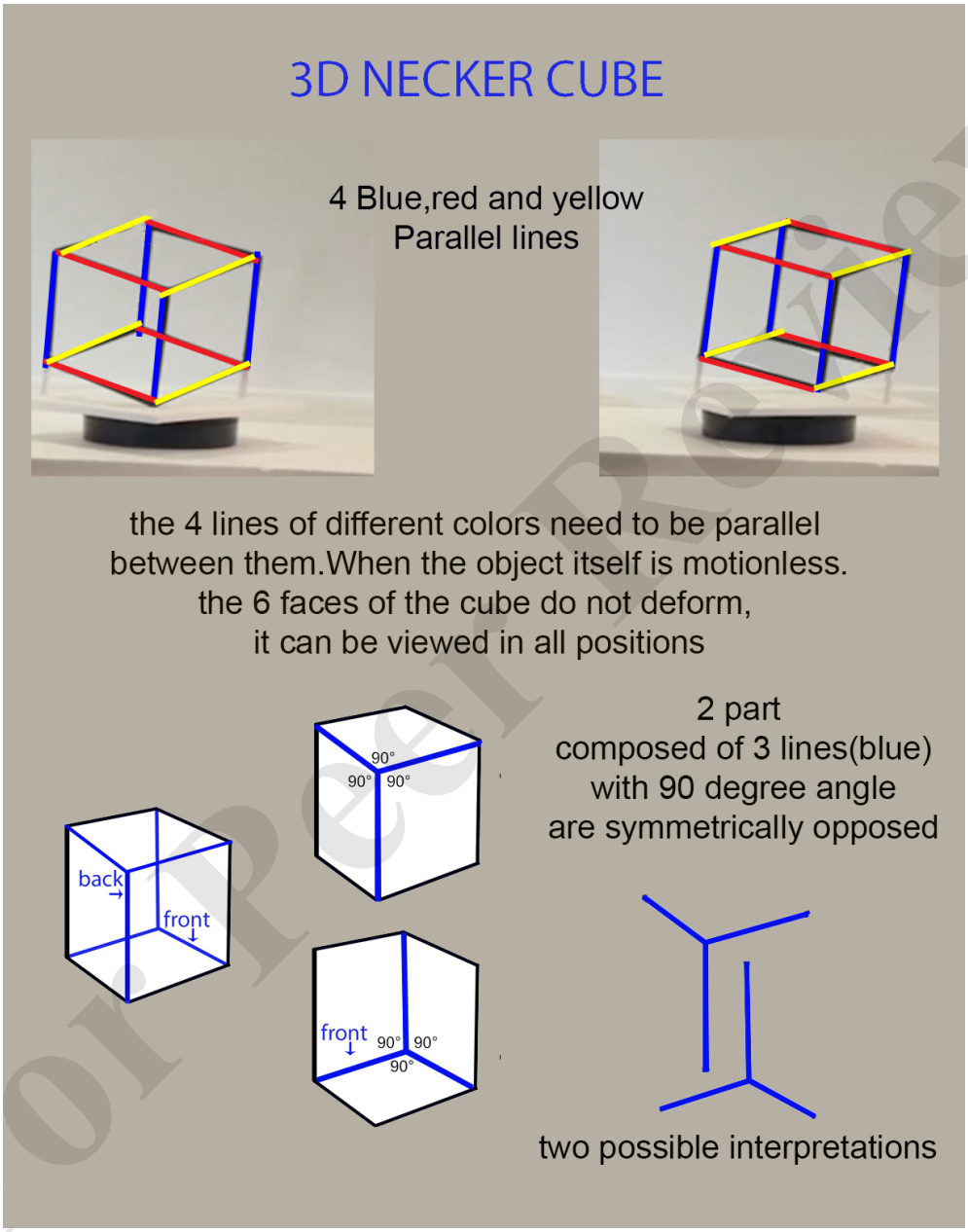
Six photographs show the model tilted into different positions. They document its appearance; a still image alone cannot demonstrate that bistability persists during movement.

Document and illustration: Olivier Redon. Original text within the illustration retained.

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Identifying groups of edges

Original document A02 · Olivier Redon



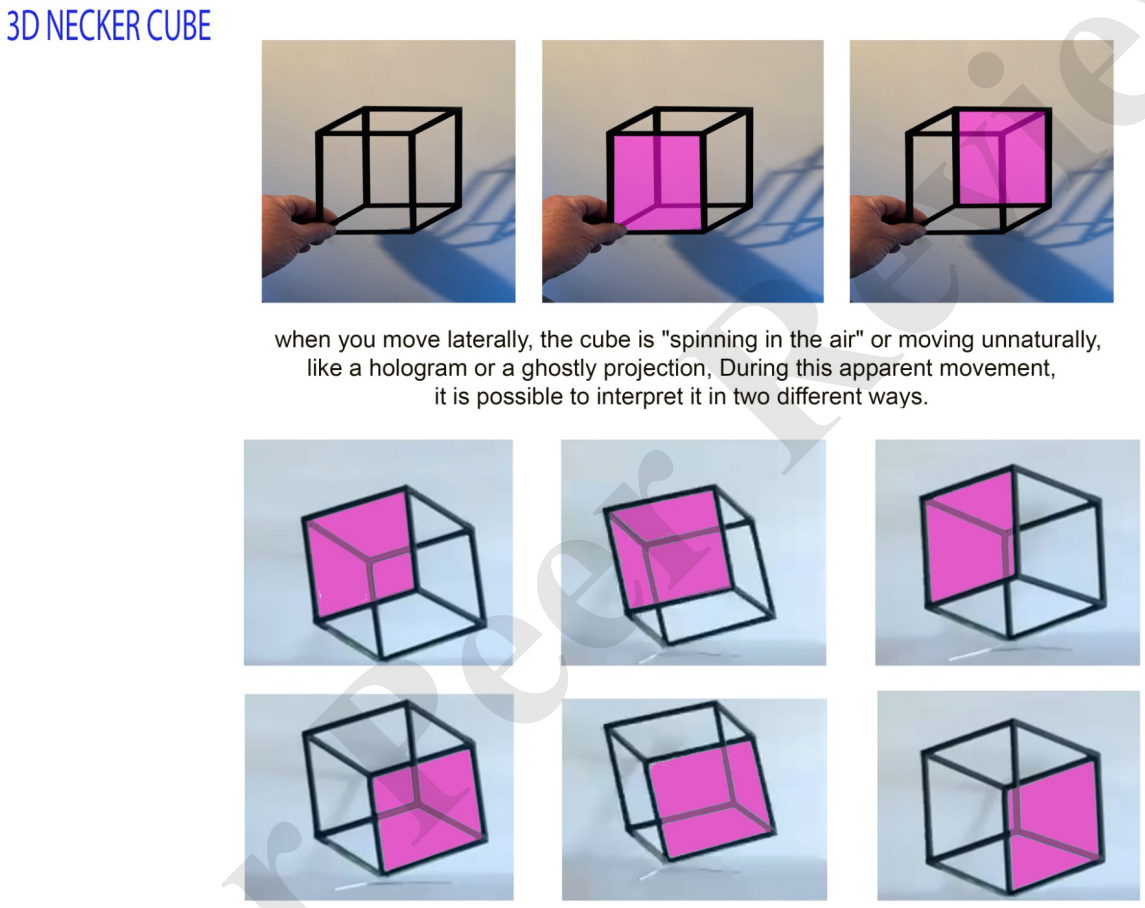
Colors group edges by direction. The lower diagram compares two arrangements of three segments. The indicated parallelism and angles express the intended geometry of this working document.

Document and illustration: Olivier Redon. Original text within the illustration retained.

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Following the pink faces

Original document A05 · Olivier Redon



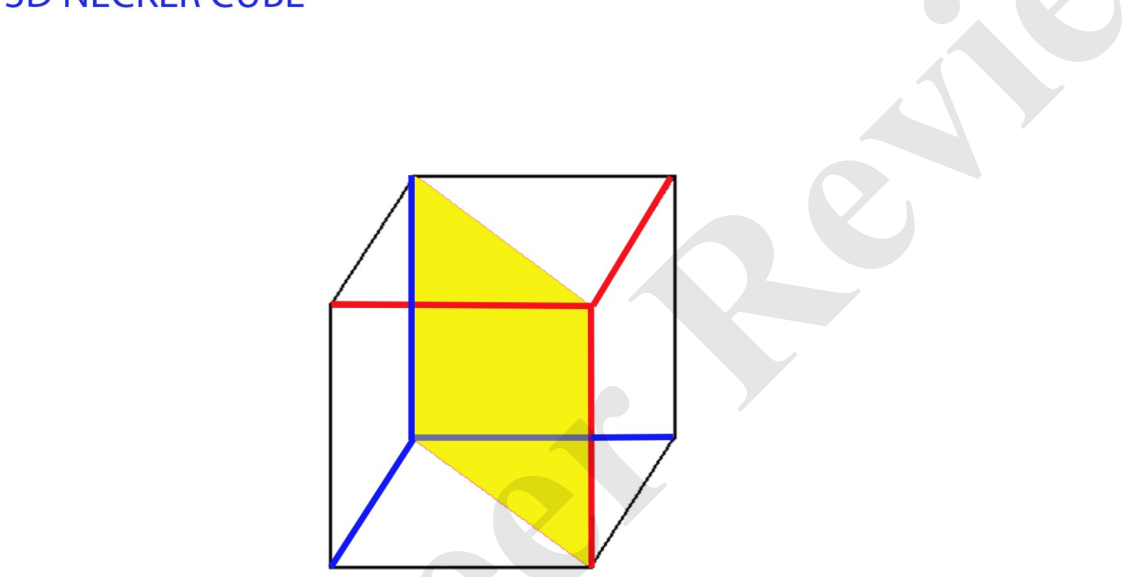
The pink areas guide the eye toward different interpretations. Compare the views without mentally changing the edges: which face seems close? The original text also describes an impression of rotation as the observer moves.

Document and illustration: Olivier Redon. Original text within the illustration retained.

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A diagonal plane as a guide

Original document A06 · Olivier Redon



Diagonal planes that are formed by cutting the cube diagonally

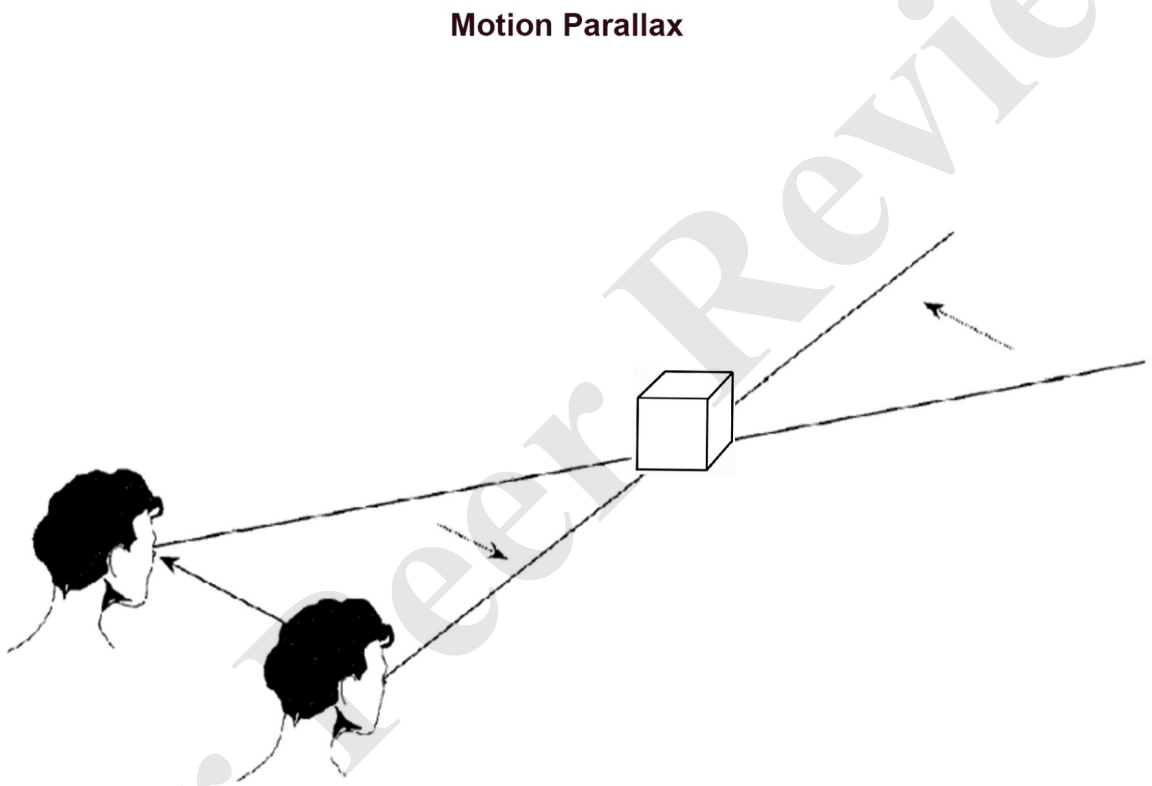
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The yellow plane cuts diagonally through the representation of the cube. It helps locate the groups of red and blue edges. A plane is an ideal geometric surface; the colored area does not imply a physical panel.

Document and illustration: Olivier Redon. Original text within the illustration retained.

When the observer moves

Original document A10 · Olivier Redon



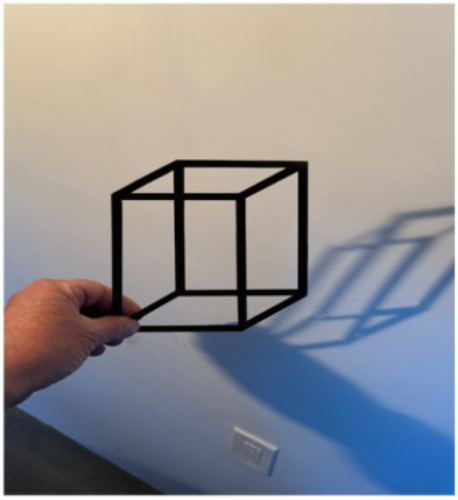
The diagram illustrates motion parallax: lines of sight change with the observer’s position. Try holding a finger in front of a wall: as you move your head, the finger appears to shift relative to the background.

Document and illustration: Olivier Redon. Original text within the illustration retained.

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From a stationary cube to apparent rotation

Document A12 · Olivier Redon



The object stays still. Its perceived depth can reverse.

Move slowly from left to right. Which face seems closest to you?

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1. Observe both interpretations

Move slowly, then stop. Try to recover the cube's other interpretation and repeat the same movement. Does the shape remain cubical in both cases? This is what I seek to achieve with this model.

2. Relate the image to parallax

As you move, nearby points shift more in the image than distant ones. Yet in the reversed interpretation, what is far away may seem close. Movement therefore provides cues that contradict the depth you perceive.

3. Separate observation from explanation

The cube may seem to rotate while remaining still. One possible explanation is that vision attributes rotation to it to maintain the reversed depth interpretation. This hypothesis still needs to be tested on my model.

Several videos are available on YouTube at these links:
https://youtu.be/zTM70eyMoa8 · https://youtu.be/5GqrPwxFrfw